

ИДЗ 4 Функциональные ряды

Задача 1. Построить область интегрирования. Изменить порядок интегрирования.

$$1.1. \int_{-2}^{-1} dy \int_{-\sqrt{2+y}}^0 f dx + \int_{-1}^0 dy \int_{-\sqrt{-y}}^0 f dx.$$

$$1.3. \int_0^1 dy \int_0^y f dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f dx.$$

$$1.5. \int_{-\sqrt{2}}^{-1} dx \int_{-\sqrt{2-x^2}}^0 f dy + \int_{-1}^0 dx \int_x^0 f dy.$$

$$1.7. \int_{-2}^{-1} dy \int_0^{\sqrt{2+y}} f dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f dx.$$

$$1.9. \int_{-\sqrt{2}}^{-1} dx \int_0^{\sqrt{2-x^2}} f dy + \int_{-1}^0 dx \int_0^{x^2} f dy.$$

$$1.11. \int_0^1 dx \int_{1-x^2}^1 f dy + \int_1^e dx \int_{\ln x}^1 f dy.$$

$$1.13. \int_0^{\pi/4} dy \int_0^{\sin y} f dx + \int_{\pi/4}^{\pi/2} dy \int_0^{\cos y} f dx.$$

$$1.15. \int_0^1 dy \int_0^{\sqrt{y}} f dx + \int_1^e dy \int_{\ln y}^1 f dx.$$

$$1.2. \int_0^1 dy \int_{-\sqrt{y}}^0 f dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{-y}}^0 f dx.$$

$$1.4. \int_0^1 dy \int_0^{\sqrt{y}} f dx + \int_1^2 dy \int_0^{\sqrt{2-y}} f dx.$$

$$1.6. \int_0^{1/\sqrt{2}} dy \int_0^{\arcsin y} f dx + \int_{1/\sqrt{2}}^1 dy \int_0^{\arccos y} f dx.$$

$$1.8. \int_0^1 dy \int_{-\sqrt{y}}^0 f dx + \int_1^e dy \int_{-1}^{-\ln y} f dx.$$

$$1.10. \int_{-2}^{-\sqrt{3}} dx \int_{-\sqrt{4-x^2}}^0 f dy + \int_{-\sqrt{3}}^0 dx \int_{\sqrt{4-x^2}-2}^0 f dy.$$

$$1.12. \int_0^1 dy \int_0^{\sqrt[3]{y}} f dx + \int_1^2 dy \int_0^{2-y} f dx.$$

$$1.14. \int_{-2}^{-1} dx \int_{-(2+x)}^0 f dy + \int_{-1}^0 dx \int_{\sqrt[3]{x}}^0 f dy.$$

$$1.16. \int_0^1 dy \int_{-\sqrt{y}}^0 f dx + \int_1^2 dy \int_{-\sqrt{2-y}}^0 f dx.$$

$$1.17. \int_0^1 dy \int_{-y}^0 f dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^0 f dx.$$

$$1.18. \int_0^1 dy \int_0^{y^2} f dx + \int_1^2 dy \int_0^{2-y} f dx.$$

$$1.19. \int_0^{\sqrt{3}} dx \int_{\sqrt{4-x^2}-2}^0 f dy + \int_{\sqrt{3}}^2 dx \int_{-\sqrt{4-x^2}}^0 f dy.$$

$$1.20. \int_{-2}^{-1} dy \int_{-(2+y)}^0 f dx + \int_{-1}^0 dy \int_{\sqrt[3]{y}}^0 f dx.$$

$$1.21. \int_0^1 dy \int_0^y f dx + \int_1^e dy \int_{\ln y}^1 f dx.$$

$$1.22. \int_0^1 dx \int_0^{x^2} f dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f dy.$$

$$1.23. \int_0^{\pi/4} dx \int_0^{\sin x} f dy + \int_{\pi/4}^{\pi/2} dx \int_0^{\cos x} f dy.$$

$$1.24. \int_{-\sqrt{2}}^{-1} dy \int_{-\sqrt{2-y^2}}^0 f dx + \int_{-1}^0 dy \int_y^0 f dx.$$

$$1.25. \int_0^1 dx \int_0^{x^2} f dy + \int_1^2 dx \int_0^{2-x} f dy.$$

$$1.26. \int_0^{\sqrt{3}} dx \int_0^{2-\sqrt{4-x^2}} f dy + \int_{\sqrt{3}}^2 dx \int_0^{\sqrt{4-x^2}} f dy.$$

$$1.27. \int_0^1 dx \int_{-\sqrt{x}}^0 f dy + \int_1^2 dx \int_{-\sqrt{2-x}}^0 f dy.$$

$$1.28. \int_0^1 dx \int_0^x f dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f dy.$$

$$1.29. \int_0^1 dy \int_0^{\sqrt{y}} f dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f dx.$$

$$1.30. \int_0^1 dx \int_0^{\sqrt{x}} f dy + \int_1^2 dx \int_0^{\sqrt{2-x}} f dy.$$

$$1.31. \int_{-2}^{-\sqrt{3}} dx \int_0^{\sqrt{4-x^2}} f dy + \int_{-\sqrt{3}}^0 dx \int_0^{2-\sqrt{4-x^2}} f dy.$$

Задача 2. Построить область интегрирования, вычислить двойной интеграл.

2.1. $\iint_D (12x^2 y^2 + 16x^3 y^3) dx dy;$ $D: x=1, y=x^2, y=-\sqrt{x}.$	2.2. $\iint_D (9x^2 y^2 + 48x^3 y^3) dx dy;$ $D: x=1, y=\sqrt{x}, y=-x^2.$
2.3. $\iint_D (36x^2 y^2 - 96x^3 y^3) dx dy;$ $D: x=1, y=\sqrt[3]{x}, y=-x^3.$	2.4. $\iint_D (18x^2 y^2 + 32x^3 y^3) dx dy;$ $D: x=1, y=x^3, y=-\sqrt[3]{x}.$
2.5. $\iint_D (27x^2 y^2 + 48x^3 y^3) dx dy;$ $D: x=1, y=x^2, y=-\sqrt[3]{x}.$	2.6. $\iint_D (18x^2 y^2 + 32x^3 y^3) dx dy;$ $D: x=1, y=\sqrt[3]{x}, y=-x^2.$
2.7. $\iint_D (18x^2 y^2 + 32x^3 y^3) dx dy;$ $D: x=1, y=x^3, y=-\sqrt{x}.$	2.8. $\iint_D (27x^2 y^2 + 48x^3 y^3) dx dy;$ $D: x=1, y=\sqrt{x}, y=-x^3.$

2.9. $\iint_D (4xy + 3x^2y^2) dx dy;$ $D: x=1, y=x^2, y=-\sqrt{x}.$	2.10. $\iint_D (12xy + 9x^2y^2) dx dy;$ $D: x=1, y=\sqrt{x}, y=-x^2.$
2.11. $\iint_D (8xy + 9x^2y^2) dx dy;$ $D: x=1, y=\sqrt[3]{x}, y=-x^3.$	2.12. $\iint_D (24xy + 18x^2y^2) dx dy;$ $D: x=1, y=x^3, y=-\sqrt[3]{x}.$
2.13. $\iint_D (12xy + 27x^2y^2) dx dy;$ $D: x=1, y=x^2, y=-\sqrt[3]{x}.$	2.14. $\iint_D (8xy + 18x^2y^2) dx dy;$ $D: x=1, y=\sqrt[3]{x}, y=-x^2.$
2.15. $\iint_D \left(\frac{4}{5}xy + \frac{9}{11}x^2y^2 \right) dx dy;$ $D: x=1, y=x^3, y=-\sqrt{x}.$	2.16. $\iint_D \left(\frac{4}{5}xy + 9x^2y^2 \right) dx dy;$ $D: x=1, y=\sqrt{x}, y=-x^3.$
2.17. $\iint_D (24xy - 48x^3y^3) dx dy;$ $D: x=1, y=x^2, y=-\sqrt{x}.$	2.18. $\iint_D (6xy + 24x^3y^3) dx dy;$ $D: x=1, y=\sqrt{x}, y=-x^2.$
2.19. $\iint_D (4xy + 16x^3y^3) dx dy;$ $D: x=1, y=\sqrt[3]{x}, y=-x^3.$	2.20. $\iint_D (4xy + 16x^3y^3) dx dy;$ $D: x=1, y=x^3, y=-\sqrt[3]{x}.$
2.21. $\iint_D (44xy + 16x^3y^3) dx dy;$ $D: x=1, y=x^2, y=-\sqrt[3]{x}.$	2.22. $\iint_D (4xy + 176x^3y^3) dx dy;$ $D: x=1, y=\sqrt[3]{x}, y=-x^3.$
2.23. $\iint_D (xy - 4x^3y^3) dx dy;$ $D: x=1, y=x^3, y=-\sqrt{x}.$	2.24. $\iint_D (4xy + 176x^3y^3) dx dy;$ $D: x=1, y=\sqrt{x}, y=-x^3.$
2.25. $\iint_D \left(6x^2y^2 + \frac{25}{3}x^4y^4 \right) dx dy;$ $D: x=1, y=x^2, y=-\sqrt{x}.$	2.26. $\iint_D (9x^2y^2 + 25x^4y^4) dx dy;$ $D: x=1, y=\sqrt{x}, y=-x^2.$
2.27. $\iint_D \left(3x^2y^2 + \frac{50}{3}x^4y^4 \right) dx dy;$ $D: x=1, y=\sqrt[3]{x}, y=-x^3.$	2.28. $\iint_D (9x^2y^2 + 25x^4y^4) dx dy;$ $D: x=1, y=x^3, y=-\sqrt[3]{x}.$
2.29. $\iint_D (54x^2y^2 + 150x^4y^4) dx dy;$ $D: x=1, y=x^2, y=-\sqrt[3]{x}.$	2.30. $\iint_D (xy - 9x^5y^5) dx dy;$ $D: x=1, y=\sqrt[3]{x}, y=-x^2.$

$$2.31. \iint_D (54x^2y^2 + 150x^4y^4) dx dy;$$

$$D: x=1, y=x^3, y=-\sqrt{x}.$$

Задача 3. Построить область интегрирования. Вычислить интеграл.

3.1. $\iint_D ye^{xy/2} dx dy;$ $D: y = \ln 2, y = \ln 3, x = 2, x = 4.$	3.2. $\iint_D y^2 \sin \frac{xy}{2} dx dy;$ $D: x = 0, y = \sqrt{\pi}, y = \frac{x}{2}.$
3.3. $\iint_D y \cos xy dx dy;$ $D: y = \pi/2, y = \pi, x = 1, x = 2.$	3.4. $\iint_D y^2 e^{-xy/4} dx dy;$ $D: x = 0, y = 2, y = x.$
3.5. $\iint_D y \sin xy dx dy;$ $D: y = \pi/2, y = \pi, x = 1, x = 2.$	3.6. $\iint_D y^2 \cos \frac{xy}{2} dx dy;$ $D: x = 0, y = \sqrt{\pi/2}, y = x/2.$
3.7. $\iint_D 4ye^{2xy} dx dy;$ $D: y = \ln 3, y = \ln 4, x = \frac{1}{2}, x = 1.$	3.8. $\iint_D 4y^2 \sin xy dx dy;$ $D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = x.$
3.9. $\iint_D y \cos 2xy dx dy;$ $D: y = \frac{\pi}{2}, y = \pi, x = \frac{1}{2}, x = 1.$	3.10. $\iint_D y^2 e^{-xy/8} dx dy;$ $D: x = 0, y = 2, y = \frac{x}{2}.$
3.11. $\iint_D 12y \sin 2xy dx dy;$ $D: y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 2, x = 3.$	3.12. $\iint_D y^2 \cos xy dx dy;$ $D: x = 0, y = \sqrt{\pi}, y = x.$
3.13. $\iint_D ye^{xy/4} dx dy;$ $D: y = \ln 2, y = \ln 3, x = 4, x = 8.$	3.14. $\iint_D y^2 \sin 2xy dx dy;$ $D: x = 0, y = \sqrt{2\pi}, y = 2x.$
3.15. $\iint_D 2y \cos 2xy dx dy;$ $D: y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 1, x = 2.$	3.16. $\iint_D y^2 e^{-xy/2} dx dy;$ $D: x = 0, y = \sqrt{2}, y = x.$

<p>3.17. $\iint_D y \sin xy \, dx dy;$ $D: y = \pi, y = 2\pi, x = \frac{1}{2}, x = 1.$</p>	<p>3.18. $\iint_D y^2 \cos 2xy \, dx dy;$ $D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = \frac{x}{2}.$</p>
<p>3.19. $\iint_D 8ye^{4xy} \, dx dy;$ $D: y = \ln 3, y = \ln 4, x = \frac{1}{4}, x = \frac{1}{2}.$</p>	<p>3.20. $\iint_D 3y^2 \sin \frac{xy}{2} \, dx dy;$ $D: x = 0, y = \sqrt{\frac{4\pi}{3}}, y = \frac{2}{3}x.$</p>
<p>3.21. $\iint_D y \cos xy \, dx dy;$ $D: y = \pi, y = 3\pi, x = 1/2, x = 1.$</p>	<p>3.22. $\iint_D y^2 e^{-xy/2} \, dx dy;$ $D: x = 0, y = 1, y = \frac{x}{2}.$</p>
<p>3.23. $\iint_D y \sin 2xy \, dx dy;$ $D: y = \pi/2, y = 3\pi/2, x = 1/2, x = 2.$</p>	<p>3.24. $\iint_D y^2 \cos xy \, dx dy;$ $D: x = 0, y = \sqrt{\pi}, y = 2x.$</p>
<p>3.25. $\iint_D 6ye^{xy/3} \, dx dy;$ $D: y = \ln 2, y = \ln 3, x = 3, x = 6.$</p>	<p>3.26. $\iint_D y^2 \sin \frac{xy}{2} \, dx dy;$ $D: x = 0, y = \sqrt{\pi}, y = x.$</p>
<p>3.27. $\iint_D y \cos 2xy \, dx dy;$ $D: y = \pi/2, y = 3\pi/2, x = 1/2, x = 2.$</p>	<p>3.28. $\iint_D y^2 e^{-xy/8} \, dx dy;$ $D: x = 0, y = 4, y = 2x.$</p>
<p>3.29. $\iint_D 3y \sin xy \, dx dy;$ $D: y = \pi/2, y = 3\pi, x = 1, x = 3.$</p>	<p>3.30. $\iint_D y^2 \cos \frac{xy}{2} \, dx dy;$ $D: x = 0, y = \sqrt{2\pi}, y = 2x.$</p>

3.31. $\iint_D 12ye^{6xy} \, dx dy;$
 $D: y = \ln 3, y = \ln 4, x = 1/6, x = 1/3.$

Задача 4. Построить область интегрирования. Вычислить интеграл.

<p>4.1. $\iiint_V 2y^2 e^{xy} \, dx \, dy \, dz;$ $V \begin{cases} x = 0, y = 1, y = x, \\ z = 0, z = 1. \end{cases}$</p>	<p>4.2. $\iiint_V x^2 z \sin(xyz) \, dx \, dy \, dz;$ $V \begin{cases} x = 2, y = \pi, z = 1, \\ x = 0, y = 1, z = 0. \end{cases}$</p>
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$\iiint_V y^2 \operatorname{ch}(2xy) \, dx \, dy \, dz;$ <p>4.3. $V \begin{cases} x = 0, y = -2, y = 4x, \\ z = 0, z = 2. \end{cases}$</p>	$\iiint_V 8y^2 z e^{2xyz} \, dx \, dy \, dz;$ <p>4.4. $V \begin{cases} x = -1, y = 2, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$</p>
$\iiint_V x^2 \operatorname{sh}(3xy) \, dx \, dy \, dz;$ <p>4.5. $V \begin{cases} x = 1, y = 2x, y = 0, \\ z = 0, z = 36. \end{cases}$</p>	$\iiint_V y^2 z \cos(xyz) \, dx \, dy \, dz;$ <p>4.6. $V \begin{cases} x = 1, y = 2\pi, z = 2, \\ x = 0, y = 1, z = 0. \end{cases}$</p>
$\iiint_V y^2 \cos\left(\frac{\pi}{4}xy\right) \, dx \, dy \, dz;$ <p>4.7. $V \begin{cases} x = 0, y = -1, y = x/2, \\ z = 0, z = -\pi^2. \end{cases}$</p>	$\iiint_V x^2 z \sin\frac{xyz}{4} \, dx \, dy \, dz;$ <p>4.8. $V \begin{cases} x = 1, y = 2\pi, z = 4, \\ x = 0, y = 0, z = 0. \end{cases}$</p>
$\iiint_V y^2 e^{-xy} \, dx \, dy \, dz;$ <p>4.9. $V \begin{cases} x = 0, y = -2, y = 4x, \\ z = 0, z = 1. \end{cases}$</p>	$\iiint_V 2y^2 z e^{2xyz} \, dx \, dy \, dz;$ <p>4.10. $V \begin{cases} x = 1, y = 1, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$</p>
$\iiint_V y^2 \operatorname{ch}(2xy) \, dx \, dy \, dz;$ <p>4.11. $V \begin{cases} x = 0, y = 1, y = x, \\ z = 0, z = 8. \end{cases}$</p>	$\iiint_V x^2 z \operatorname{sh}(xyz) \, dx \, dy \, dz;$ <p>4.12. $V \begin{cases} x = 2, y = 1, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$</p>
$\iiint_V y^2 e^{xy/2} \, dx \, dy \, dz;$ <p>4.13. $V \begin{cases} x = 0, y = 2, y = 2x, \\ z = 0, z = -1. \end{cases}$</p>	$\iiint_V y^2 z \cos\frac{xyz}{3} \, dx \, dy \, dz;$ <p>4.14. $V \begin{cases} x = 3, y = 1, z = 2\pi, \\ x = 0, y = 0, z = 0. \end{cases}$</p>
$\iiint_V y^2 \cos\left(\frac{\pi xy}{2}\right) \, dx \, dy \, dz;$ <p>4.15. $V \begin{cases} x = 0, y = -1, y = x, \\ z = 0, z = 2\pi^2. \end{cases}$</p>	$\iiint_V x^2 z \operatorname{sh}(xyz) \, dx \, dy \, dz;$ <p>4.16. $V \begin{cases} x = 1, y = -1, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$</p>
$\iiint_V y^2 \cos(\pi xy) \, dx \, dy \, dz;$ <p>4.17. $V \begin{cases} x = 0, y = 1, y = 2x, \\ z = 0, z = \pi^2. \end{cases}$</p>	$\iiint_V 2x^2 z \operatorname{sh}(2xyz) \, dx \, dy \, dz;$ <p>4.18. $V \begin{cases} x = 2, y = 1/2, z = 1/2, \\ x = 0, y = 0, z = 0. \end{cases}$</p>

$\iiint_V x^2 \operatorname{sh}(2xy) \, dx \, dy \, dz;$ $4.19. \quad V \begin{cases} x = -1, y = x, y = 0, \\ z = 0, z = 8. \end{cases}$	$\iiint_V x^2 z \sin \frac{xyz}{2} \, dx \, dy \, dz;$ $4.20. \quad V \begin{cases} x = 1, y = 4, z = \pi, \\ x = 0, y = 0, z = 0. \end{cases}$
$\iiint_V y^2 \operatorname{ch}(xy) \, dx \, dy \, dz;$ $4.21. \quad V \begin{cases} x = 0, y = -1, y = x, \\ z = 0, z = 2. \end{cases}$	$\iiint_V x^2 z \operatorname{ch}(xyz) \, dx \, dy \, dz;$ $4.22. \quad V \begin{cases} x = 1, y = 1, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$
$\iiint_V x^2 \cos\left(\frac{\pi}{2}xy\right) \, dx \, dy \, dz;$ $4.23. \quad V \begin{cases} x = 2, y = x, y = 0, \\ z = 0, z = \pi. \end{cases}$	$\iiint_V y^2 z \cos \frac{xyz}{9} \, dx \, dy \, dz;$ $4.24. \quad V \begin{cases} x = 9, y = 1, z = 2\pi, \\ x = 0, y = 0, z = 0. \end{cases}$
$\iiint_V x^2 \cos(\pi xy) \, dx \, dy \, dz;$ $4.25. \quad V \begin{cases} x = 1, y = 2x, y = 0, \\ z = 0, z = 4\pi. \end{cases}$	$\iiint_V y^2 z \operatorname{ch}\left(\frac{xyz}{2}\right) \, dx \, dy \, dz;$ $4.26. \quad V \begin{cases} x = 2, y = -1, z = 2, \\ x = 0, y = 0, z = 0. \end{cases}$
$\iiint_V y^2 \operatorname{ch}(3xy) \, dx \, dy \, dz;$ $4.27. \quad V \begin{cases} x = 0, y = 2, y = 6x, \\ z = 0, z = -3. \end{cases}$	$\iiint_V 2y^2 z \operatorname{ch}(2xyz) \, dx \, dy \, dz;$ $4.28. \quad V \begin{cases} x = \frac{1}{2}, y = 2, z = -1, \\ x = 0, y = 0, z = 0. \end{cases}$
$\iiint_V x^2 \sin(4\pi xy) \, dx \, dy \, dz;$ $4.29. \quad V \begin{cases} x = 1, y = x/2, y = 0, \\ z = 0, z = 8\pi. \end{cases}$	$\iiint_V 8y^2 z e^{-xyz} \, dx \, dy \, dz;$ $4.30. \quad V \begin{cases} x = 2, y = -1, z = 2, \\ x = 0, y = 0, z = 0. \end{cases}$

$$\iiint_V x^2 \operatorname{sh}(xy) \, dx \, dy \, dz;$$

$$4.31. \quad V \begin{cases} x = 2, y = x/2, y = 0, \\ z = 0, z = 1. \end{cases}$$

Задача 5. Найти площадь фигуры, ограниченной данными линиями (7).

$y^2 - 2y + x^2 = 0,$ 5.1. $y^2 - 4y + x^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$	$x^2 - 4x + y^2 = 0,$ 5.2. $x^2 - 8x + y^2 = 0,$ $y = 0, y = x/\sqrt{3}.$
$y^2 - 6y + x^2 = 0,$ 5.3. $y^2 - 8y + x^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$	$x^2 - 2x + y^2 = 0,$ 5.4. $x^2 - 4x + y^2 = 0,$ $y = 0, y = x.$
$y^2 - 8y + x^2 = 0,$ 5.5. $y^2 - 10y + x^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$	$x^2 - 4x + y^2 = 0,$ 5.6. $x^2 - 8x + y^2 = 0,$ $y = 0, y = x.$
$y^2 - 4y + x^2 = 0,$ 5.7. $y^2 - 6y + x^2 = 0,$ $y = x, x = 0.$	$x^2 - 2x + y^2 = 0,$ 5.8. $x^2 - 10x + y^2 = 0,$ $y = 0, y = \sqrt{3}x.$
$y^2 - 6y + x^2 = 0,$ 5.9. $y^2 - 10y + x^2 = 0,$ $y = x, x = 0.$	$x^2 - 2x + y^2 = 0,$ 5.10. $x^2 - 4x + y^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$
$y^2 - 2y + x^2 = 0,$ 5.11. $y^2 - 4y + x^2 = 0,$ $y = \sqrt{3}x, x = 0.$	$x^2 - 2x + y^2 = 0,$ 5.12. $x^2 - 6x + y^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$
$y^2 - 4y + x^2 = 0,$ 5.13. $y^2 - 6y + x^2 = 0,$ $y = \sqrt{3}x, x = 0.$	$x^2 - 2x + y^2 = 0,$ 5.14. $x^2 - 8x + y^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$
$y^2 - 2y + x^2 = 0,$ 5.15. $y^2 - 6y + x^2 = 0,$ $y = x/\sqrt{3}, y = 0.$	$x^2 - 2x + y^2 = 0,$ 5.16. $x^2 - 4x + y^2 = 0,$ $y = 0, y = x/\sqrt{3}.$
$y^2 - 2y + x^2 = 0,$ 5.17. $y^2 - 10y + x^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$	$x^2 - 2x + y^2 = 0,$ 5.18. $x^2 - 6x + y^2 = 0,$ $y = 0, y = x/\sqrt{3}.$

$y^2 - 4y + x^2 = 0,$ 5.19. $y^2 - 10y + x^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$	$x^2 - 2x + y^2 = 0,$ 5.20. $x^2 - 6x + y^2 = 0,$ $y = 0, y = x.$
$y^2 - 2y + x^2 = 0,$ 5.21. $y^2 - 4y + x^2 = 0,$ $y = x, x = 0.$	$x^2 - 2x + y^2 = 0,$ 5.22. $x^2 - 4x + y^2 = 0,$ $y = 0, y = \sqrt{3}x.$
$y^2 - 6y + x^2 = 0,$ 5.23. $y^2 - 8y + x^2 = 0,$ $y = x, x = 0.$	$x^2 - 4x + y^2 = 0,$ 5.24. $x^2 - 8x + y^2 = 0,$ $y = 0, y = \sqrt{3}x.$
$y^2 - 4y + x^2 = 0,$ 5.25. $y^2 - 8y + x^2 = 0,$ $y = x, x = 0.$	$x^2 - 4x + y^2 = 0,$ 5.26. $x^2 - 8x + y^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$
$y^2 - 4y + x^2 = 0,$ 5.27. $y^2 - 8y + x^2 = 0,$ $y = \sqrt{3}x, x = 0.$	$x^2 - 4x + y^2 = 0,$ 5.28. $x^2 - 6x + y^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$
$y^2 - 2y + x^2 = 0,$ 5.29. $y^2 - 10y + x^2 = 0,$ $y = x/\sqrt{3}, x = 0.$	$x^2 - 6x + y^2 = 0,$ 5.30. $x^2 - 10x + y^2 = 0,$ $y = x/\sqrt{3}, y = \sqrt{3}x.$
$y^2 - 4y + x^2 = 0,$ 5.31. $y^2 - 8y + x^2 = 0,$ $y = x/\sqrt{3}, x = 0.$	

Задача 6. Пластинка D задана неравенствами, μ - поверхностная плотность. Построить изображение пластины. Найти массу пластинки. (9)

6.1. $D: x^2 + y^2/4 \leq 1;$ $\mu = y^2.$	$D: 1 \leq x^2/9 + y^2/4 \leq 2;$ 6.2. $y \geq 0, y \leq \frac{2}{3}x;$ $\mu = y/x.$
6.3. $D: x^2/9 + y^2/25 \leq 1;$ $y \geq 0;$ $\mu = x^2 y.$	6.4. $D: x^2/9 + y^2/25 \leq 1;$ $y \geq 0;$ $\mu = 7x^2 y/18.$

6.5. $D: 1 \leq x^2/9 + y^2/4 \leq 4;$ $y \geq 0, y \leq x/2;$ $\mu = 8y/x^3.$	6.6. $D: x^2/9 + y^2 \leq 1;$ $x \geq 0;$ $\mu = 7xy^6.$
6.7. $D: x^2/4 + y^2 \leq 1;$ $\mu = 4y^4.$	6.8. $D: 1 \leq x^2/4 + y^2/9 \leq 4;$ $x \geq 0, y \leq 3x/2;$ $\mu = x/y.$
6.9. $D: 1 \leq x^2/16 + y^2/4 \leq 4;$ $x \geq 0, y \leq x/2;$ $\mu = x/y.$	6.10. $D: x^2/4 + y^2/9 \leq 1;$ $x \geq 0, y \geq 0;$ $\mu = x^3y.$
6.11. $D: x^2/4 + y^2 \leq 1;$ $x \geq 0, y \geq 0;$ $\mu = 6x^3y^3.$	6.12. $D: 1 \leq x^2/4 + y^2 \leq 25;$ $x \geq 0, y \leq x/2;$ $\mu = x/y^3.$
6.13. $D: x^2/9 + y^2/4 \leq 1;$ $\mu = x^2y^2.$	6.14. $D: x^2/16 + y^2 \leq 1;$ $x \geq 0, y \geq 0;$ $\mu = 5xy^7.$
6.15. $D: x^2/4 + y^2 \leq 1;$ $x \geq 0, y \geq 0;$ $\mu = 30x^3y^7.$	6.16. $D: 1 \leq x^2/9 + y^2/4 \leq 3;$ $y \geq 0, y \leq \frac{2}{3}x;$ $\mu = y/x.$
6.17. $D: x^2 + y^2/25 \leq 1;$ $y \geq 0;$ $\mu = 7x^4y.$	6.18. $D: x^2 + y^2/9 \leq 1;$ $y \geq 0;$ $\mu = 35x^4y^3.$
6.19. $D: x^2/4 + y^2/9 \leq 1;$ $\mu = x^2.$	6.20. $D: 1 \leq x^2 + y^2/16 \leq 9;$ $y \geq 0, y \leq 4x;$ $\mu = y/x^3.$
6.21. $D: x^2/9 + y^2 \leq 1;$ $x \geq 0;$ $\mu = 11xy^8.$	6.22. $D: 1 \leq x^2/4 + y^2/16 \leq 5;$ $x \geq 0, y \leq 2x;$ $\mu = x/y.$
6.23. $D: 1 \leq x^2/9 + y^2/4 \leq 5;$ $x \geq 0, y \leq 2x/3;$ $\mu = x/y.$	6.24. $D: x^2/4 + y^2/9 \leq 1;$ $x \geq 0, y \geq 0;$ $\mu = x^5y.$

6.25.	$D: x^2/4 + y^2/25 \leq 1;$ $\mu = x^4.$	6.26.	$D: x^2 + y^2/4 \leq 1;$ $x \geq 0, y \geq 0;$ $\mu = 15x^5 y^3.$
6.27.	$D: 1 \leq x^2/4 + y^2/9 \leq 36;$ $x \geq 0, y \geq \frac{3}{2}x;$ $\mu = 9x/y^3.$	6.28.	$D: x^2/100 + y^2 \leq 1;$ $x \geq 0, y \geq 0;$ $\mu = 6xy^9.$
6.29.	$D: x^2/16 + y^2 \leq 1;$ $x \geq 0, y \geq 0;$ $\mu = 105x^3 y^9.$	6.30.	$D: 1 \leq x^2/9 + y^2/16 \leq 2;$ $y \geq 0, y \leq \frac{4}{3}x;$ $\mu = 27y/x^5.$

6.31. $D: 1 \leq x^2/16 + y^2 \leq 3;$
 $x \geq 0, y \geq x/4;$
 $\mu = x/y^5.$

Задача 7. Найти объем тела, заданного ограничивающими его поверхностями. (11)

7.1.	$x^2 + y^2 = 2y,$ $z = 5/4 - x^2, z = 0.$	7.2.	$x^2 + y^2 = y, x^2 + y^2 = 4y,$ $z = \sqrt{x^2 + y^2}, z = 0.$
7.3.	$x^2 + y^2 = 8\sqrt{2}x,$ $z = x^2 + y^2 - 64,$ $z = 0 (z \geq 0).$	7.4.	$x^2 + y^2 + 4x = 0,$ $z = 8 - y^2, z = 0.$
7.5.	$x^2 + y^2 = 6x, x^2 + y^2 = 9x,$ $z = \sqrt{x^2 + y^2}, z = 0,$ $y = 0 (y \leq 0)$	7.6.	$x^2 + y^2 = 6\sqrt{2}y,$ $z = x^2 + y^2 - 36,$ $z = 0 (z \geq 0).$
7.7.	$x^2 + y^2 = 2y,$ $z = 9/4 - x^2, z = 0.$	7.8.	$x^2 + y^2 = 2y, x^2 + y^2 = 5y,$ $z = \sqrt{x^2 + y^2}, z = 0.$
7.9.	$x^2 + y^2 + 2\sqrt{2}y = 0,$ $z = x^2 + y^2 - 4,$ $z = 0 (z \geq 0).$	7.10.	$x^2 + y^2 = 4x,$ $z = 10 - y^2, z = 0.$

$x^2 + y^2 = 7x, \quad x^2 + y^2 = 9x,$ 7.11. $z = \sqrt{x^2 + y^2}, \quad z = 0,$ $y = 0 \quad (y \leq 0)$	$x^2 + y^2 = 8\sqrt{2}y,$ 7.12. $z = x^2 + y^2 - 64,$ $z = 0 \quad (z \geq 0).$
$x^2 + y^2 = 2y,$ 7.13. $z = 13/4 - x^2, \quad z = 0.$	$x^2 + y^2 = 3y, \quad x^2 + y^2 = 6y,$ 7.14. $z = \sqrt{x^2 + y^2}, \quad z = 0.$
$x^2 + y^2 = 6\sqrt{2}x,$ 7.15. $z = x^2 + y^2 - 36,$ $z = 0 \quad (z \geq 0).$	$x^2 + y^2 = 2\sqrt{2}y,$ 7.16. $z = x^2 + y^2 - 4,$ $z = 0 \quad (z \geq 0).$
$x^2 + y^2 = 4x,$ 7.17. $z = 12 - y^2, \quad z = 0.$	$x^2 + y^2 = 8x, \quad x^2 + y^2 = 11x,$ 7.18. $z = \sqrt{x^2 + y^2}, \quad z = 0,$ $y = 0 \quad (y \leq 0)$
$x^2 + y^2 = 4\sqrt{2}x,$ 7.19. $z = x^2 + y^2 - 16,$ $z = 0 \quad (z \geq 0).$	$x^2 + y^2 = 4y,$ 7.20. $z = 4 - x^2, \quad z = 0.$
$x^2 + y^2 = 4y, \quad x^2 + y^2 = 7y,$ 7.21. $z = \sqrt{x^2 + y^2}, \quad z = 0.$	$x^2 + y^2 = 4\sqrt{2}y,$ 7.22. $z = x^2 + y^2 - 16,$ $z = 0 \quad (z \geq 0).$
$x^2 + y^2 + 2x = 0,$ 7.23. $z = 17/4 - y^2, \quad z = 0.$	$x^2 + y^2 = 9x, \quad x^2 + y^2 = 12x,$ 7.24. $z = \sqrt{x^2 + y^2}, \quad z = 0,$ $y = 0 \quad (y \geq 0)$
$x^2 + y^2 + 2\sqrt{2}x = 0,$ 7.25. $z = x^2 + y^2 - 4,$ $z = 0 \quad (z \geq 0).$	$x^2 + y^2 = 4y,$ 7.26. $z = 6 - x^2, \quad z = 0.$
$x^2 + y^2 = 10x, \quad x^2 + y^2 = 13x,$ 7.27. $z = \sqrt{x^2 + y^2}, \quad z = 0,$ $y = 0 \quad (y \geq 0)$	$x^2 + y^2 = 2\sqrt{2}x,$ 7.28. $z = x^2 + y^2 - 4,$ $z = 0 \quad (z \geq 0).$
$x^2 + y^2 = 2x,$ 7.29. $z = 21/4 - y^2, \quad z = 0.$	$x^2 + y^2 = 5y, \quad x^2 + y^2 = 8y,$ 7.30. $z = \sqrt{x^2 + y^2}, \quad z = 0.$

$$7.31. \quad \begin{aligned} x^2 + y^2 + 2x &= 0, \\ z &= 25/4 - y^2, \quad z = 0. \end{aligned}$$

Задача 8. Найти объем тела, заданного ограничивающими его поверхностями. (13)

8.1. $\begin{aligned} z &= \sqrt{9 - x^2 - y^2}, \\ 9z/2 &= x^2 + y^2. \end{aligned}$	8.2. $\begin{aligned} z &= 15\sqrt{x^2 + y^2}/2, \\ z &= 17/2 - x^2 - y^2. \end{aligned}$
8.3. $\begin{aligned} z &= \sqrt{4 - x^2 - y^2}, \\ z &= \sqrt{(x^2 + y^2)}/255. \end{aligned}$	8.4. $\begin{aligned} z &= \sqrt{64 - x^2 - y^2}, \quad z = 1, \\ x^2 + y^2 &= 60 \\ &(\text{внутри цилиндра}). \end{aligned}$
8.5. $\begin{aligned} z &= \sqrt{\frac{16}{9} - x^2 - y^2}, \\ 2z &= x^2 + y^2. \end{aligned}$	8.6. $\begin{aligned} z &= 3\sqrt{x^2 + y^2}, \\ z &= 10 - x^2 - y^2. \end{aligned}$
8.7. $\begin{aligned} z &= \sqrt{25 - x^2 - y^2}, \\ z &= \sqrt{(x^2 + y^2)}/99. \end{aligned}$	8.8. $\begin{aligned} z &= \sqrt{100 - x^2 - y^2}, \quad z = 6, \\ x^2 + y^2 &= 51 \\ &(\text{внутри цилиндра}). \end{aligned}$
8.9. $\begin{aligned} z &= 21\sqrt{x^2 + y^2}/2, \\ z &= 23/2 - x^2 - y^2. \end{aligned}$	8.10. $\begin{aligned} z &= \sqrt{16 - x^2 - y^2}, \\ 6z &= x^2 + y^2. \end{aligned}$
8.11. $\begin{aligned} z &= \sqrt{9 - x^2 - y^2}, \\ z &= \sqrt{(x^2 + y^2)}/80. \end{aligned}$	8.12. $\begin{aligned} z &= \sqrt{81 - x^2 - y^2}, \quad z = 5, \\ x^2 + y^2 &= 45 \\ &(\text{внутри цилиндра}). \end{aligned}$
8.13. $\begin{aligned} z &= \sqrt{1 - x^2 - y^2}, \\ 3z/2 &= x^2 + y^2. \end{aligned}$	8.14. $\begin{aligned} z &= 6\sqrt{x^2 + y^2}, \\ z &= 16 - x^2 - y^2. \end{aligned}$
8.15. $\begin{aligned} z &= \sqrt{36 - x^2 - y^2}, \\ z &= \sqrt{(x^2 + y^2)}/63. \end{aligned}$	8.16. $\begin{aligned} z &= \sqrt{64 - x^2 - y^2}, \quad z = 4, \\ x^2 + y^2 &= 39 \\ &(\text{внутри цилиндра}). \end{aligned}$
8.17. $\begin{aligned} z &= \sqrt{144 - x^2 - y^2}, \\ 18z &= x^2 + y^2. \end{aligned}$	8.18. $\begin{aligned} z &= 3\sqrt{x^2 + y^2}/2, \\ z &= 5/2 - x^2 - y^2. \end{aligned}$

8.19. $z = \sqrt{9 - x^2 - y^2},$ $z = \sqrt{(x^2 + y^2)}/35.$	8.20. $z = \sqrt{49 - x^2 - y^2}, z = 3,$ $x^2 + y^2 = 33$ (внутри цилиндра).
8.21. $z = \sqrt{36 - x^2 - y^2},$ $9z = x^2 + y^2.$	8.22. $z = 9\sqrt{x^2 + y^2},$ $z = 22 - x^2 - y^2.$
8.23. $z = \sqrt{16 - x^2 - y^2},$ $z = \sqrt{(x^2 + y^2)}/15.$	8.24. $x^2 + y^2 = 27$ (внутри цилиндра).
8.25. $z = \sqrt{4/9 - x^2 - y^2},$ $z = x^2 + y^2.$	8.26. $z = 12\sqrt{x^2 + y^2},$ $z = 28 - x^2 - y^2.$
8.27. $z = \sqrt{9 - x^2 - y^2},$ $z = \sqrt{(x^2 + y^2)}/8.$	8.28. $x^2 + y^2 = 21$ (внутри цилиндра).
8.29. $z = \sqrt{64 - x^2 - y^2},$ $12z = x^2 + y^2.$	8.30. $z = 9\sqrt{x^2 + y^2}/2,$ $z = 11/2 - x^2 - y^2.$

8.31.
 $z = \sqrt{36 - x^2 - y^2},$
 $z = \sqrt{(x^2 + y^2)}/3.$

Задача 9. Тело V задано ограничивающими его поверхностями, μ - плотность. Найти массу тела. (16)

9.1. $64(x^2 + y^2) = z^2, x^2 + y^2 = 4,$ $y = 0, z = 0 (y \geq 0, z \geq 0),$ $\mu = 5(x^2 + y^2)/4.$	9.2. $x^2 + y^2 + z^2 = 4, x^2 + y^2 = 1,$ $(x^2 + y^2 \leq 1), x = 0 (x \geq 0);$ $\mu = 4 z .$
9.3. $x^2 + y^2 = 1, x^2 + y^2 = 2z,$ $x = 0, y = 0, z = 0 (x \geq 0, y \geq 0);$ $\mu = 10x.$	9.4. $x^2 + y^2 = \frac{16}{49}z^2, x^2 + y^2 = \frac{4}{7}z,$ $x = 0, y = 0, (x \geq 0, y \geq 0);$ $\mu = 80yz.$

$x^2 + y^2 + z^2 = 1, \quad x^2 + y^2 = 4z^2,$ 9.5. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$ $\mu = 20z.$	$36(x^2 + y^2) = z^2, \quad x^2 + y^2 = 1,$ 9.6. $x = 0, z = 0 (x \geq 0, z \geq 0),$ $\mu = \frac{5}{6}(x^2 + y^2).$
$x^2 + y^2 + z^2 = 16, \quad x^2 + y^2 = 4,$ 9.7. $(x^2 + y^2 \leq 4);$ $\mu = 2 z .$	$x^2 + y^2 = 4, \quad x^2 + y^2 = 8z,$ 9.8. $x = 0, y = 0, z = 0 (x \geq 0, y \geq 0);$ $\mu = 5x.$
$x^2 + y^2 = \frac{4}{25}z^2, \quad x^2 + y^2 = \frac{2}{5}z,$ 9.9. $x = 0, y = 0, (x \geq 0, y \geq 0);$ $\mu = 28xz.$	$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = z^2,$ 9.10. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$ $\mu = 6z.$
$25(x^2 + y^2) = z^2, \quad x^2 + y^2 = 4,$ $x = 0, y = 0, z = 0$ 9.11. $(x \geq 0, y \geq 0, z \geq 0),$ $\mu = 2(x^2 + y^2).$	$x^2 + y^2 + z^2 = 9, \quad x^2 + y^2 = 4,$ 9.12. $(x^2 + y^2 \leq 4), y = 0 (y \geq 0);$ $\mu = z .$
$x^2 + y^2 = 1, \quad x^2 + y^2 = 6z,$ 9.13. $x = 0, y = 0, z = 0 (x \geq 0, y \geq 0);$ $\mu = 90y.$	$x^2 + y^2 = \frac{1}{25}z^2, \quad x^2 + y^2 = \frac{1}{5}z,$ 9.14. $x = 0, y = 0, (x \geq 0, y \geq 0);$ $\mu = 14yz.$
$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 9z^2,$ 9.15. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$ $\mu = 10z.$	$9(x^2 + y^2) = z^2, \quad x^2 + y^2 = 4,$ $x = 0, y = 0, z = 0$ 9.16. $(x \geq 0, y \geq 0, z \geq 0),$ $\mu = 5(x^2 + y^2)/3.$
$x^2 + y^2 + z^2 = 4,$ 9.17. $x^2 + y^2 = 1, (x^2 + y^2 \leq 1);$ $\mu = z .$	$x^2 + y^2 = 1, \quad x^2 + y^2 = z,$ $x = 0, y = 0, z = 0,$ 9.18. $(x \geq 0, y \geq 0);$ $\mu = 10y.$

$x^2 + y^2 = \frac{1}{49}z^2, \quad x^2 + y^2 = \frac{1}{7}z,$ <p>9.19. $x = 0, y = 0, (x \geq 0, y \geq 0);$ $\mu = 10xz.$</p>	$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 4z^2,$ <p>9.20. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$ $\mu = 10z.$</p>
$16(x^2 + y^2) = z^2, \quad x^2 + y^2 = 1,$ <p>9.21. $x = 0, y = 0, z = 0 \quad (x \geq 0, y \geq 0, z \geq 0),$ $\mu = 5(x^2 + y^2).$</p>	$x^2 + y^2 + z^2 = 16,$ <p>9.22. $x^2 + y^2 = 4 \quad (x^2 + y^2 \leq 4);$ $\mu = z .$</p>
$x^2 + y^2 = 4, \quad x^2 + y^2 = 4z,$ <p>9.23. $x = 0, y = 0, z = 0 \quad (x \geq 0, y \geq 0);$ $\mu = 5y.$</p>	$x^2 + y^2 = z^2, \quad x^2 + y^2 = z,$ <p>9.24. $x = 0, y = 0, (x \geq 0, y \geq 0);$ $\mu = 35yz.$</p>
$x^2 + y^2 + z^2 = 1, \quad x^2 + y^2 = z^2,$ <p>9.25. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$ $\mu = 32z.$</p>	$x^2 + y^2 = z^2, \quad x^2 + y^2 = 4,$ <p>9.26. $x = 0, y = 0, z = 0$ $(x \geq 0, y \geq 0, z \geq 0),$ $\mu = 5(x^2 + y^2)/2.$</p>
$x^2 + y^2 + z^2 = 9, \quad x^2 + y^2 = 4,$ <p>9.27. $(x^2 + y^2 \leq 4), \quad z = 0 \quad (z \geq 0);$ $\mu = 2z.$</p>	$x^2 + y^2 = 1, \quad x^2 + y^2 = 3z,$ <p>9.28. $x = 0, y = 0, z = 0$ $(x \geq 0, y \geq 0);$ $\mu = 15x.$</p>
$x^2 + y^2 = \frac{4}{49}z^2, \quad x^2 + y^2 = \frac{2}{7}z,$ <p>9.29. $x = 0, y = 0, (x \geq 0, y \geq 0);$ $\mu = 20xz.$</p>	$x^2 + y^2 + z^2 = 16, \quad x^2 + y^2 = 9z^2,$ <p>9.30. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$ $\mu = 5z.$</p>

$$4(x^2 + y^2) = z^2, \quad x^2 + y^2 = 1,$$

9.31. $y = 0, z = 0 \quad (y \geq 0, z \geq 0),$
 $\mu = 10(x^2 + y^2).$